



Oxford Cambridge and RSA

Wednesday 8 June 2022 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

- 1 (a) Find a vector which is perpendicular to both $3\mathbf{i} - 5\mathbf{j} - \mathbf{k}$ and $\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$. [1]

The equations of two lines are $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 11\mathbf{j} - 4\mathbf{k} + \mu(-\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$.

- (b) Show that the lines intersect, stating the point of intersection. [5]

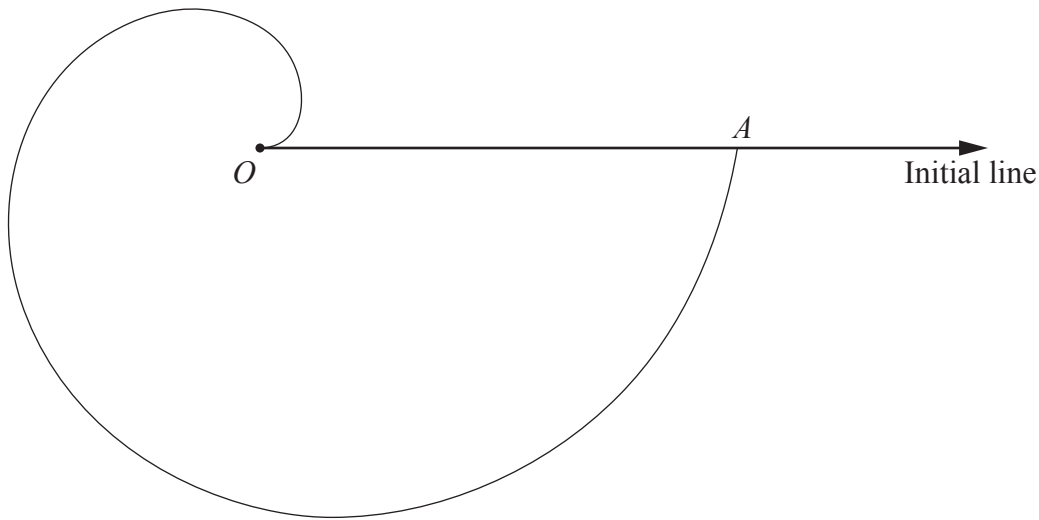
- 2 Two polar curves, C_1 and C_2 , are defined by $C_1: r = 2\theta$ and $C_2: r = \theta + 1$ where $0 \leq \theta \leq 2\pi$.

C_1 intersects the initial line at two points, the pole and the point A .

- (a) Write down the polar coordinates of A . [2]

- (b) Determine the polar coordinates of the point of intersection of C_1 and C_2 . [2]

The diagram below shows a sketch of C_1 .



- (c) On the copy of this sketch in the Printed Answer Booklet, sketch C_2 . [1]

- 3 **In this question you must show detailed reasoning.**

The roots of the equation $4x^3 + 6x^2 - 3x + 9 = 0$ are α , β and γ .

Find a cubic equation with integer coefficients whose roots are $\alpha + \beta$, $\beta + \gamma$ and $\gamma + \alpha$. [6]

4 In this question you must show detailed reasoning.

Determine the smallest value of n for which $\frac{1^2 + 2^2 + \dots + n^2}{1 + 2 + \dots + n} > 341$. [4]

5 (a) By using the exponential definitions of $\sinh x$ and $\cosh x$, prove the identity $\cosh 2x \equiv \cosh^2 x + \sinh^2 x$. [2]

(b) Hence find an expression for $\cosh 2x$ in terms of $\cosh x$. [1]

(c) Determine the solutions of the equation $5\cosh 2x = 16\cosh x + 21$, giving your answers in exact logarithmic form. [4]

- 6 A particle, P , positioned at the origin, O , is projected with a certain velocity along the x -axis. P is then acted on by a single force which varies in such a way that P moves backwards and forwards along the x -axis.

When the time after projection is t seconds, the displacement of P from the origin is x m and its velocity is v ms⁻¹.

The motion of P is modelled using the differential equation $\ddot{x} + \omega^2 x = 0$, where ω rad s⁻¹ is a positive constant.

- (a) Write down the general solution of this differential equation. [1]

D is the point where $x = d$ for some positive constant, d . When P reaches D it comes to instantaneous rest.

- (b) Using the answer to part (a), determine expressions, in terms of ω , d and t only, for the following quantities

- x
- v

[3]

- (c) Hence show that, according to the model, $v^2 = \omega^2(d^2 - x^2)$. [1]

The quantity z is defined by $z = \frac{1}{v}$.

- (d) Using part (c), determine an expression for z_m , the mean value of z **with respect to the displacement**, as P moves directly from O to D . [2]

One measure of the validity of the model is consideration of the value of z_m . If z_m exceeds 8 then the model is considered to be valid.

The value of d is measured as 0.25 to 2 significant figures. The value of ω is measured as 0.75 ± 0.02 .

- (e) Determine what can be inferred about the validity of the model from the given information. [1]
- (f) Find, according to the model, the least possible value of the velocity with which P was initially projected. Give your answer to 2 significant figures. [2]

7 You are given that a is a parameter which can take only real values.

The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} 2 & 4 & -6 \\ -3 & 10-4a & 9 \\ 7 & 4 & 4 \end{pmatrix}$.

(a) Find an expression for the determinant of \mathbf{A} in terms of a . [2]

You are given the following system of equations in x , y and z .

$$\begin{array}{rcl} 2x + & 4y - 6z = & 6 \\ -3x + (10-4a)y + & 9z = & -9 \\ 7x + & 4y + 4z = & 11 \end{array}$$

The system can be written in the form $\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -9 \\ 11 \end{pmatrix}$.

(b) (i) In the case where \mathbf{A} is **not** singular, solve the given system of equations by using \mathbf{A}^{-1} . [5]

(ii) In the case where \mathbf{A} is singular describe the configuration of the planes whose equations are the three equations of the system. [3]

The transformation represented by \mathbf{A} is denoted by T .

A 3-D object of volume $|5a - 20|$ is transformed by T to a 3-D image.

(c) (i) Determine the range of values of a for which the orientation of the image is the reverse of the orientation of the object. [1]

(ii) Determine the range of values of a for which the volume of the image is less than the volume of the object. [2]

8 In this question you must show detailed reasoning.

It is given that $\sum_{r=k}^{98} \frac{5r+2}{r(r+1)(r+2)} = \frac{20539}{34650}$ for some k .

Determine the value of k . [7]

9 In this question you must show detailed reasoning.

(a) Show that $\operatorname{Re}(e^{4i\theta}(e^{i\theta} + e^{-i\theta})^4) = a \cos 4\theta \cos^4 \theta$, where a is an integer to be determined. [3]

(b) Hence show that $\cos \frac{1}{12}\pi = \frac{1}{2} \sqrt[4]{b + c\sqrt{3}}$, where b and c are integers to be determined. [6]

10 The coordinates of the points A and B are $(3, -2, -1)$ and $(13, 10, 9)$ respectively.

- The plane Π_A contains A and the plane Π_B contains B .
- The planes Π_A and Π_B are parallel.
- The x and y components of any normal to plane Π_A are equal.
- The shortest distance between Π_A and Π_B is 2.

There are **two** possible solution planes for Π_A which satisfy the above conditions.

Determine the acute angle between these two possible solution planes. [8]

END OF QUESTION PAPER

BLANK PAGE

OCR

Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series. If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.