

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

--	--	--	--	--

--	--	--	--

Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper
reference

9FM0/3A

Further Mathematics

Advanced

PAPER 3A: Further Pure Mathematics 1

You must have:

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

P65497A

©2022 Pearson Education Ltd.

Q:1/1/1/1/



Pearson

1. An ellipse has equation $\frac{x^2}{16} + \frac{y^2}{4} = 1$ and eccentricity e_1

A hyperbola has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and eccentricity e_2

Given that $e_1 \times e_2 = 1$

(a) show that $a^2 = 3b^2$

(4)

Given also that the coordinates of the foci of the ellipse are the same as the coordinates of the foci of the hyperbola,

(b) determine the equation of the hyperbola.

(3)



Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 1 is 7 marks)



P 6 5 4 9 7 A 0 3 3 2

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

Handwriting practice area with horizontal lines.

(Total for Question 2 is 7 marks)



3. With respect to a fixed origin O , the points A and B have coordinates $(2, 2, -1)$ and $(4, 2p, 1)$ respectively, where p is a constant.

For each of the following, determine the possible values of p for which,

- (a) OB makes an angle of 45° with the positive x -axis (3)

- (b) $\vec{OA} \times \vec{OB}$ is parallel to $\begin{pmatrix} 4 \\ -p \\ 2 \end{pmatrix}$ (3)

- (c) the area of triangle OAB is $3\sqrt{2}$ (3)



DO NOT WRITE IN THIS AREA



P 6 5 4 9 7 A 0 7 3 2

Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Handwriting practice area with horizontal lines.

(Total for Question 3 is 9 marks)



This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 4 is 6 marks)



5. The rectangular hyperbola H has equation $xy = 36$

(a) Use calculus to show that the equation of the tangent to H at the point $P\left(6t, \frac{6}{t}\right)$ is

$$yt^2 + x = 12t$$

(3)

The point $Q\left(12t, \frac{3}{t}\right)$ also lies on H .

(b) Find the equation of the tangent to H at the point Q .

(2)

The tangent at P and the tangent at Q meet at the point R .

(c) Show that as t varies the locus of R is also a rectangular hyperbola.

(4)



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 6 5 4 9 7 A 0 1 3 3 2

Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 5 is 9 marks)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

6. The points P , Q and R have position vectors $\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$ respectively.
- (a) Determine a vector equation of the plane that passes through the points P , Q and R , giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, where λ and μ are scalar parameters. (2)
- (b) Determine the coordinates of the point of intersection of the plane with the x -axis. (4)



DO NOT WRITE IN THIS AREA



P 6 5 4 9 7 A 0 1 7 3 2

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 6 is 6 marks)



7.

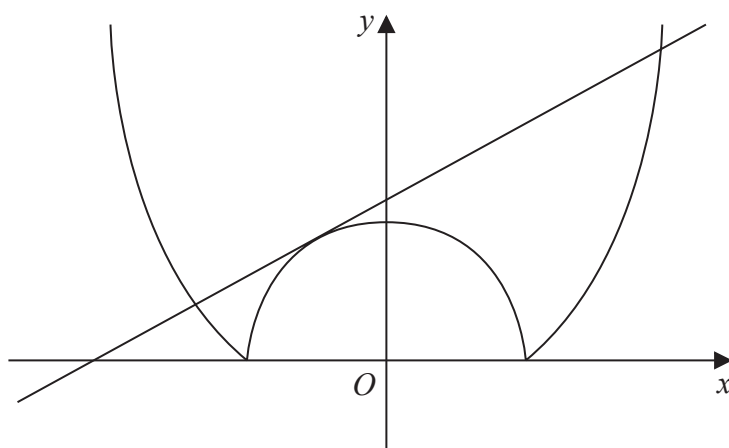


Figure 1

Figure 1 shows a sketch of the curve with equation $y = |x^2 - 8|$ and a sketch of the straight line with equation $y = mx + c$, where m and c are positive constants.

The equation

$$|x^2 - 8| = mx + c$$

has exactly 3 roots, as shown in Figure 1.

(a) Show that

$$m^2 - 4c + 32 = 0 \quad (2)$$

Given that $c = 3m$

(b) determine the value of m and the value of c (3)

(c) Hence solve

$$|x^2 - 8| \geq mx + c \quad (3)$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Lined area for writing the answer to Question 7.



P 6 5 4 9 7 A 0 2 1 3 2

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA



8.

$$\left[\begin{array}{l} \text{The Taylor series expansion of } f(x) \text{ about } x = a \text{ is given by} \\ f(x) = f(a) + (x - a)f'(a) + \frac{(x - a)^2}{2!}f''(a) + \dots + \frac{(x - a)^r}{r!}f^{(r)}(a) + \dots \end{array} \right]$$

- (i) (a) Use differentiation to determine the Taylor series expansion of $\ln x$, in ascending powers of $(x - 1)$, up to and including the term in $(x - 1)^2$ (4)
- (b) Hence prove that

$$\lim_{x \rightarrow 1} \left(\frac{\ln x}{x - 1} \right) = 1 \quad (2)$$

- (ii) Use L'Hospital's rule to determine

$$\lim_{x \rightarrow 0} \left(\frac{1}{(x + 3)\tan(6x)\operatorname{cosec}(2x)} \right)$$

(Solutions relying entirely on calculator technology are not acceptable.) (4)



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Lined area for writing the answer to Question 8.



P 6 5 4 9 7 A 0 2 5 3 2

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

Handwriting practice area with horizontal lines.

(Total for Question 8 is 10 marks)



9. A particle P moves along a straight line.

At time t minutes, the displacement, x metres, of P from a fixed point O on the line is modelled by the differential equation

$$t^2 \frac{d^2 x}{dt^2} - 2t \frac{dx}{dt} + 2x + 16t^2 x = 4t^3 \sin 2t \quad (\text{I})$$

(a) Show that the transformation $x = ty$ transforms equation (I) into the equation

$$\frac{d^2 y}{dt^2} + 16y = 4 \sin 2t \quad (5)$$

(b) Hence find a general solution for the displacement of P from O at time t minutes.

(8)



DO NOT WRITE IN THIS AREA



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



DO NOT WRITE IN THIS AREA



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

